**Answer to the Question No. - 1**

To show that ) using Definition 2, we proceed as follows:

**Ω-notation:**

If there exist constants c > 0 and n0 ≥ 0 such that:

for all n ≥ n0.

Here, we need to show:

for sufficiently large n (for some n0)

Let,

So, we need to show that:

Dividing both sides by , we get,

For choosing constants c and n0,

We can let as a positive constant and we need to choose n0 large enough such that the equation holds for all n ≥ n0.

As the right side is constant, let’s analyze the left side.

Since, , the term grows much faster than . Because logarithmic functions grow much slower than any polynomial of n.

The above proof shows that,

The condition for Ω-notation is satisfied because,

for all n ≥ n0.

So, by Definition 2, it is proven that,

)

**Answer to the Question No. – 2**

If we convert the given time into microseconds we get,

1 second = microseconds

1 minute = microseconds

1 hour = microseconds

1 day = microseconds

1 month = microseconds

1 year = microseconds

1 century = microseconds

Let the time = t, then for each expression the result will be as following:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  | *t* |
|  |  | approximations by calculating |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  | approximations by calculating |

The value for the comparison of running times has been shown in the following table:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 second | 1 minute | 1 hour | 1 day | 1 month | 1 year | 1 century |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  | 1000 | 7745 | 60000 | 293938 |  |  |  |
|  | 100 | 391 | 1532 | 4420 | 13736 | 31593 | 146645 |
|  | 19 | 25 | 31 | 36 | 41 | 44 | 51 |
|  | 9 | 11 | 12 | 13 | 15 | 16 | 17 |

**Answer to the Question No. – 3**

Assuming that are constants, we need to find the relative asymptotic growths for each pair of expressions (A, B).

**vs :**

Since grows much faster than any power of , the limit goes to 0. We get,

So, it will be **yes** for O, o but **no** for Ω, , *.*

**vs :**

Using logarithms,

Since, grows faster than ,

= 0

So, it will be **yes** for O, o but **no** for Ω, , *.*

**vs :**

The value for is in the range between -1 to +1 and it doesn’t have a consistent growth pattern. As there is no consistency, none of the asymptotic relations can be justified. So, it will be **no** for each one.

**vs :**

If we take the ratio, it would be , which tends to . It refers to the lower bound. So, it will be **yes** for Ω, but **no** for O, o and *.*

**vs:**

We can rewrite the expressions as,

=

=

Therefore,

=

As both the expressions are equal, they will grow at the same rate. So, it will be **yes** for O, Ω and but **no** for o and *.*

**vs:**

Both the functions represent the same output. As the difference is almost non-existent, they grow at the same rate. So, for this case, it will be **yes** for O, Ω and but **no** for o and *.*

Considering all the cases, the final results have been shown in the following table:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| A | B | O | o | Ω |  |  |
|  |  | Yes | Yes | No | No | No |
|  |  | Yes | Yes | No | No | No |
|  |  | No | No | No | No | No |
|  |  | No | No | Yes | Yes | No |
|  |  | Yes | No | Yes | No | Yes |
|  |  | Yes | No | Yes | No | Yes |